Student number: ______________________

GIRRAWEEN HIGH SCHOOL

2021 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION,

MATHEMATICS EXTENSION 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- NESA approved calculators may be used.
- A reference sheet is provided at the back of this paper.
- In section II, Show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 60 marks

- Attempt all questions
- Allow about 1 hour and 45 minutes for this section

SECTION 1

10 marks

Attempt questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1 - 10

1. Given that $\underset{\sim}{x} = 5\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{y} = -2\underset{\sim}{i} - 5\underset{\sim}{j}$. The magnitude and direction of $\underset{\sim}{x} + \underset{\sim}{y}$ is

(A) 3.6 ; 326°

(B) 3.6 ; 34°

(C) 3.6 ; 146°

(D) 3.6 ; 214°

2. In the expansion of $(2x+k)^6$, the coefficients of x and x^2 are equal. What is the value of k?

(A) 5 (B) 6 (C) 11 (D) 12

3. The coefficient of x^{-5} in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{20}$ is

(A) -77520 **(B)** -155040 **(C)** -248064 **(D)** -496128

4. The domain and inverse of $f(x)=4\log_e(x+3)-2$ are

(A) $x>3;\quad y=e^{\frac{x+2}{4}}-3$

(B) $x>-3;\ y=e^{\frac{x+2}{4}}-2$

(C) $x>-3;\ y=e^{\frac{x+2}{4}}-3$

(D) $x>3;\quad y=e^{\frac{x+2}{4}}-2$

5. Consider the parametric equation $x=5\cos\theta-2$ and $y=5\sin\theta+3$. Which of these is the corresponding cartesian equation?

(A) $x^2-4x+y^2-6y=12$

(B) $x^2+4x+y^2+6y=12$

(C) $x^2-4x+y^2+6y=12$

(D) $x^2+4x+y^2-6y=12$

6. What is the derivative of $y=\cos^{-1}\left(\frac{x}{4}\right)$

(A) $-\frac{1}{\sqrt{16-x^2}}$ **(B)** $-\frac{2}{\sqrt{16-x^2}}$

(C) $-\frac{4}{\sqrt{16-x^2}}$ **(D)** $-\frac{6}{\sqrt{16-x^2}}$

7. What is the domain and range of $f(x)=2\sin^{-1}\left(\dfrac{x}{2}\right)$?

(A) $D:-2\le x\le 2,\ \mathrm{R}:-\dfrac{\pi}{2}\le y\le\dfrac{\pi}{2}$

(B) $D:-2\le x\le 2,\ R:-\pi\le y\le\pi$

(C) $D:-\dfrac{1}{2}\le x\le\dfrac{1}{2},\ R:-\dfrac{\pi}{2}\le \mathrm{y}\le\dfrac{\pi}{2}$

(D) $D:-\dfrac{1}{2}\le x\le\dfrac{1}{2}, R:-\pi\le y\le\pi$

8. $\int \sin^2 3x\,dx$ is equal to which of the following?

(A) $\dfrac{x}{2}-\dfrac{\sin 6x}{3}+C$

(B) $\dfrac{x}{2}-\dfrac{\sin 6x}{6}+C$

(C) $\dfrac{x}{2}-\dfrac{\sin 6x}{9}+C$

(D) $\dfrac{x}{2}-\dfrac{\sin 6x}{12}+C$

9. What is the value of k such that $\displaystyle\int_0^k \frac{dx}{1+(x-1)^2}=\frac{\pi}{2}$

(A) $2\sqrt{3}$ **(B)** $\sqrt{3}$ **(C)** 2 **(D)** 1

10. Which of the following is a factor of $2x^4-4x^3-10x^2+12x$?

(A) $x+1$ **(B)** $x-2$ **(C)** $x-3$ **(D)** $x+4$

Section II

60 marks

Attempt all questions

Allow about 1 hour and 45 minutes for this section

Start each question on a new page in the answer booklet provided.

Your responses should include relevant mathematical reasoning and /or calculations. Extra writing space is available on request.

Question 11 (12 marks) **Marks**

(a) Solve $\frac{6}{5x-2} \le 2$ 3

(b) Prove that $\cot 2x + \cot x = \frac{\sin 3x}{\sin 2x \sin x}$ 2

(c) Use the substitution $u = \ln 3x$, to find $\int \frac{dx}{x(\ln 3x)^2}$ 3

(d) Let $f(x) = \frac{2x}{\sqrt{1-x^2}}$

(i) For what values of x is $f(x)$ undefined? 1

(ii) Find $\int_0^{\frac{1}{2}} \frac{2x\,dx}{\sqrt{1-x^2}}$ using the substitution $x = \sin u$. 3

End of Question 11

Question 12 (12 marks)

(a) (i) Express $5\sin x + 12\cos x$ in the form $A\sin(x+\alpha)$ where $0 \le \alpha \le \frac{\pi}{2}$ (Give the value of α in radians, correct to 2 decimal places) 3

(ii) Hence solve $5\sin x + 12\cos x = 8$ for $0 \le x \le \pi$ (Give the value or values of x in radians correct to 2 decimal places) 2

(b) Six people attend a dinner party.

(i) In how many different ways can they be arranged around a round table? 1

(ii) In how many different ways can they be arranged if a particular couple must sit together? 1

(iii) What is the probability that, if the people are seated at random, the couple are sitting apart from each other? 1

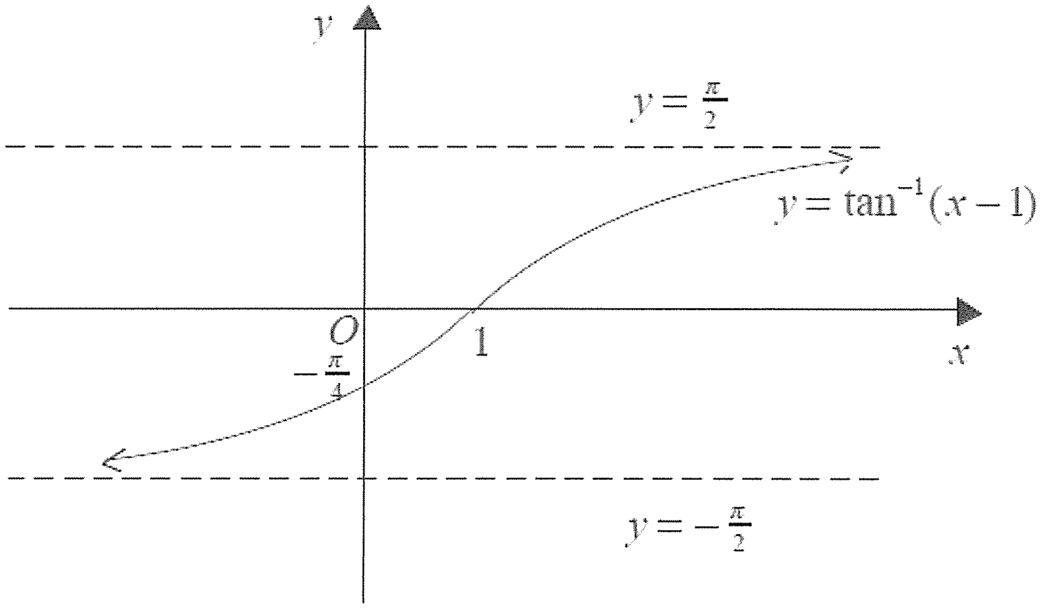
(c) Use mathematical induction to prove that

$(1^2+1)\,1! + (2^2+1)\,2! + (3^2+1)\,3! + \ldots + (n^2+1)\,n! = n\,(n+1)!$ for all positive integers $n \ge 1$. 4

End of Question12

Question 13 (12 marks)

(a)



The region in the first quadrant bounded by the curve $y=\tan^{-1}(x-1)$ and the y – axis between the lines $y=0$ and $y=\dfrac{\pi}{4}$ is rotated through one complete revolution about the y – axis.

(i) Show that the volume V of the solid of revolution is given by

$$V=\pi\int_0^{\frac{\pi}{4}}(1+\tan y)^2\,dy.$$ 1

(ii) Hence find the value of V in simplest exact form. 3

(b) A particle is projected from a point O with velocity V m/s at an angle θ to the horizontal. At any time t seconds the horizontal and vertical components of displacement are given by

$x = Vt\cos\theta$ and $y = Vt\sin\theta - \frac{1}{2}gt^2$ where g is the acceleration due to gravity.

Show that the cartesian equation of the path is given by $y = x\tan\theta - \frac{gx^2}{2V^2}\left(1+\tan^2\theta\right)$ 2

(c) A particle is projected from O with velocity $60\ m/s$ at an angle α to the horizontal.

T seconds later, another particle is projected from O with velocity $60\ m/s$ at an angle β

To the horizontal where $\beta < \alpha$. The two particles collide 240 metres horizontally from O

and at a height of 80 metres above O. Taking $g = 10 m/s^2$ and using results from (a)

(i) Show that $\tan\alpha = 2$ and $\tan\beta = 1$. 3

(ii) Find the value of T in simplest exact form. 3

End of Question 13

Question 14 (12 marks)

(a) **(i)** Differentiate $y = x\cos^{-1}x - \sqrt{1-x^2}$. 2

(ii) Hence calculate the exact value of $\int_0^{\frac{1}{2}} \cos^{-1} x\,dx$ 2

(b) Solve $x^4 - 5x^3 - 9x^2 + 81x - 108 = 0$, given that $P(x) = x^4 - 5x^3 - 9x^2 + 81x - 108$

has a triple zero. 3

(c) A bottle of medicine which is initially at a temperature of $10°C$ is placed into a room which has a constant temperature of $25°C$. The medicine warms at a rate proportional to the difference between the temperature of the room and the temperature (T) of the medicine. That is, T satisfies the equation $\frac{dT}{dt} = -k(T-25)$

(i) Show that $T = 25 + Ae^{-kt}$ is a solution of this equation. 2

(ii) If the temperature of the medicine after 10 minutes is $16°C$, find its temperature after 40 minutes. 3

End of Question 14

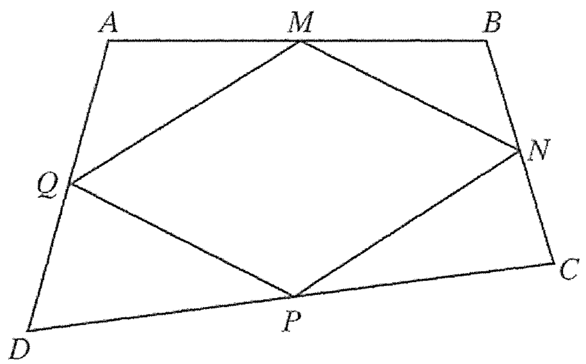
Question 15 (12 marks)

(a) For what value(s) of m are the vectors $\begin{pmatrix} 10m-17 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} m \\ 2 \end{pmatrix}$ perpendicular? 3

(b) Consider the vectors given by $\underset{\sim}{u} = b\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{w} = 2\underset{\sim}{i} + b\underset{\sim}{j}$ where b is a real number.

If the acute angle between the two vectors is $60°$, find the two possible values for b. 4

(c) Consider the quadrilateral $ABCD$. The midpoints of AB, BC, CD and DA are M, N, P and Q respectively.



Let $\overrightarrow{AB} = \underset{\sim}{a}$, $\overrightarrow{BC} = \underset{\sim}{b}$, $\overrightarrow{CD} = \underset{\sim}{c}$ and $\overrightarrow{DA} = \underset{\sim}{d}$

(i) Prove that $\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c} + \underset{\sim}{d} = 0$ 2

(ii) Hence prove that $MNPQ$ is a parallelogram. 3

END OF TEST

Year 12 Trial HSC Extension 1, 2021 Solutions

1. $\underset{\sim}{x} = 5\underset{\sim}{i} + 3\underset{\sim}{j}$

$\underset{\sim}{y} = -2\underset{\sim}{i} - 5\underset{\sim}{j}$

$\underset{\sim}{x} + \underset{\sim}{y} = 3\underset{\sim}{i} - 2\underset{\sim}{j}$

$|\underset{\sim}{x} + \underset{\sim}{y}| = \sqrt{9+4}$
$= \sqrt{13} = 3.6$

$\tan\theta = \frac{-2}{3}$

acute $\angle = \tan^{-1}\left(\frac{2}{3}\right)$
$= 34^\circ$

$\theta = 360 - 34$
$= 326$ (A)

2. ${}^6C_4 \times 2^2 \times k^4 = {}^6C_5 \times 2 \times k^5$

$k = \frac{{}^6C_4 \times 2}{{}^6C_5} = 5$ (A)

3. $T_{r+1} = {}^nC_r\, a^{n-r} b^r$

$(-1)^r\, {}^{20}C_r (2x^2)^{20-r} \left(\frac{1}{x}\right)^r$

$(-1)^r\, {}^{20}C_r\, 2^{20-r} x^{40-3r}$

$40 - 3r = -5$

$r = 15$

$(-1)^{15}\, {}^{20}C_{15}\, 2^5$

$= -496128$ (D)

4. $x + 3 > 0$

$x > -3$

$y = 4\log_e(x+3) - 2$

$x = 4\log_e(y+3) - 2$

$x + 2 = 4\log_e(y+3)$

$\frac{x+2}{4} = \log_e(y+3)$

$y + 3 = e^{\frac{x+2}{4}}$

$y = e^{\frac{x+2}{4}} - 3$ (C)

5. $x = 5\cos\theta - 2$, $y = 5\sin\theta + 3$

$\cos\theta = \frac{x+2}{5}$, $\sin\theta = \frac{y-3}{5}$

$\frac{(x+2)^2}{25} + \frac{(y-3)^2}{25} = 1$

$(x+2)^2 + (y-3)^2 = 25$

$x^2 + 4x + 4 + y^2 - 6y + 9 = 25$

$x^2 + 4x + y^2 - 6y = 12$ (D)

6. $y = \cos^{-1}\left(\frac{x}{4}\right)$

$y' = \frac{-1}{\sqrt{16 - x^2}}$ (A)

7. $f(x) = 2\sin^{-1}\left(\frac{x}{2}\right)$

$D: -2 \le x \le 2$

$R: -\pi \le y \le \pi$ (B)

8. $\int \sin^2 3x \, dx$

$= \int \frac{1-\cos 6x}{2} \, dx$

$= \frac{1}{2}\int (1-\cos 6x) \, dx$

$= \frac{1}{2}\left[x - \frac{\sin 6x}{6}\right] + C$ (D)

$= \frac{x}{2} - \frac{\sin 6x}{12} + C$

9. $\int_0^k \frac{dx}{1+(x-1)^2} = \frac{\pi}{2}$

$\left[\tan^{-1}(x-1)\right]_0^k = \frac{\pi}{2}$

$\tan^{-1}(k-1) - \tan^{-1}(-1) = \frac{\pi}{2}$

$\tan^{-1}(k-1) + \frac{\pi}{4} = \frac{\pi}{2}$

$k-1 = 1$

$k = 2$ (C)

(10) $P(x) = 2x^4 - 4x^3 - 10x^2 + 12x$

$P(3) = 2 \times 81 - 4 \times 27$

$-10 \times 9 + 36$

$= 0$

$\therefore (x-3)$ is a factor

(C)

Question 11 (12 marks)

(a) $\frac{6}{5x-2} \leq 2$

Multiply by $(5x-2)^2$, $x \neq \frac{2}{5}$

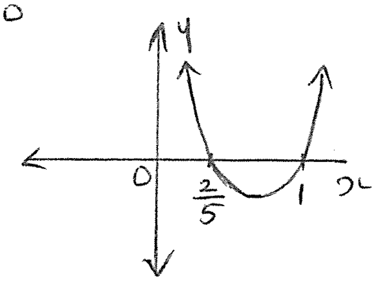
$6(5x-2) \leq 2(5x-2)^2$

$(5x-2)^2 - 3(5x-2) \geq 0$

$(5x-2)(5x-5) \geq 0$

x intercepts

$x = \frac{2}{5} = 0.4$, $x = 1$



From the graph

$(5x-2)(5x-5) \geq 0$

when $x \leq \frac{2}{5}$ or $x \geq 1$ (3)

But $x \neq \frac{2}{5}$

$\therefore x < \frac{2}{5}$ or $x \geq 1$

(b) LHS $= \cot 2x + \cot x$

$= \frac{\cos 2x}{\sin 2x} + \frac{\cos x}{\sin x}$

$= \frac{\cos 2x \sin x + \cos x \sin 2x}{\sin 2x \sin x}$ (2)

$= \frac{\sin 3x}{\sin 2x \sin x} = \text{RHS}$

(c) $u = \ln 3x$, $\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x} dx$

$\int \frac{du}{u^2} = \int u^{-2} du = \frac{-1}{u} + C$ (3)

$= \frac{-1}{\ln 3x} + C$

(d) (i) $f(x)$ is undefined when $1-x^2 \le 0$

$(1+x)(1-x) \le 0$ (1)

$x \le -1$ or $x \ge 1$

(ii) $x = \sin u$

$\frac{dx}{du} = \cos u$

when $x=0$, $0 = \sin u$

$u = \sin^{-1}(0) = 0$

when $x = \frac{1}{2}$, $\frac{1}{2} = \sin u$

$u = \sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$

$\int_0^{\frac{\pi}{6}} \frac{2\sin u \cos u \, du}{\cos u}$

$= 2\int_0^{\frac{\pi}{6}} \sin u \, du$

$= -2\left[\cos u\right]_0^{\frac{\pi}{6}}$

$= -2\left(\cos\frac{\pi}{6} - \cos 0\right)$

$= -2\left(\frac{\sqrt{3}}{2} - 1\right)$ (3)

$= \underline{\underline{2-\sqrt{3}}}$

Question 12 (12 marks)

(a) $5\sin x + 12\cos x = A\sin(x+\alpha)$

$= A\sin x\cos\alpha + A\cos x\sin\alpha$

$A\cos\alpha = 5$

$A\sin\alpha = 12$

$A^2\sin^2\alpha + A^2\cos^2\alpha = 169$

$A^2 = 169$

$A = \sqrt{169}$

$= 13 \quad (\because A > 0)$ (3)

$\sin\alpha = \frac{12}{13}$, $\cos\alpha = \frac{5}{13}$

$\alpha = 1.18^c$

$5\sin x + 12\cos x = 13\sin(x + 1.18)$

(ii) $13\sin(x+1.18) = 8$

$\sin(x+1.18) = \frac{8}{13} \quad 0 \le x \le \pi$

$u = x + 1.18$

$\sin u = \frac{8}{13} \quad 1.18 \le u \le 4.32$

$u = 0.66, \ \pi - 0.66$

$= 0.66, \ 2.48, \ 0.66+2\pi, \ 2.48+2\pi$

$= 0.66, \ 2.48, \ 6.94, \ 8.76$

$x + 1.18 = 2.48$ (2)

$\underline{\underline{x = 1.3}}$

(b)(i) $5! = 120$ ①

(ii) $4! \times 2 = 48$ ①

(iii) $\frac{120-48}{120} = \frac{72}{120} = \frac{3}{5}$ ①

(c) $(1^2+1)\times 1! + (2^2+1)\times 2! + (3^2+1)\times 3! + \cdots + (n^2+1)n! = n(n+1)!$

$n = 1$

$LHS = (1^2+1)\times 1! = 2$

$RHS = 1(1+1)! = 2$

$LHS = RHS$

$\therefore$ true for $n=1$

Assume true for $n=k$

$(1^2+1)\times 1! + (2^2+1)\times 2! + \cdots + (k^2+1)\times k! = k(k+1)!$ —— ①

To prove true for $n=k+1$

$(1^2+1)\times 1! + (2^2+1)\times 2! + \cdots + (k^2+1)\times k! + ((k+1)^2+1)\times(k+1)!$

$= (k+1)(k+1+1)!$

$= (k+1)(k+2)!$ —— ②

LHS of ②

$= (1^2+1)\times 1! + (2^2+1)\times 2! + \cdots + (k^2+1)\times k! + ((k+1)^2+1)\times(k+1)!$

$= k(k+1)! + ((k+1)^2+1)\times(k+1)!$ (by assumption ①)

$= (k+1)!\,[k + (k+1)^2 + 1]$

$= (k+1)!\,(k^2+3k+2)$

4 mark

$= (k+1)!\,(k+1)(k+2)$

$= (k+1)(k+2)! =$ RHS of ②

If the result is true for $n=k$, then it is true for $n=k+1$.
Hence by the principle of mathematical induction, the result is true for all positive integers $n \geq 1$.

Question 13 (12 marks)

(a)(i) $y = \tan^{-1}(x-1)$

$\tan y = x-1$

$x = 1+\tan y$

$V = \pi\int_0^{\frac{\pi}{4}} x^2\,dy$ ①

$= \pi\int_0^{\frac{\pi}{4}} (1+\tan y)^2\,dy$

(ii) $V = \pi\int_0^{\frac{\pi}{4}} (1+2\tan y+\tan^2 y)\,dy$

$= \pi\int_0^{\frac{\pi}{4}} \left(2\frac{\sin y}{\cos y} + \sec^2 y\right)dy$

$= \pi\left[-2\log_e(\cos y)+\tan y\right]_0^{\frac{\pi}{4}}$

$= \pi\left\{\left(-2\log_e\left(\frac{1}{\sqrt{2}}\right)+1\right) - \left(-2\log_e(1)+0\right)\right\}$

$= \pi\left(-2\log_e\left(\frac{1}{\sqrt{2}}\right)+1\right)$

$= \pi\left(\log_e\left(2^{-\frac{1}{2}}\right)^{-2}+1\right)$

$= \pi\left(\log_e 2 + 1\right)$ ③

(b) $x = Vt\cos\theta$ — ①

$y = Vt\sin\theta - \frac{1}{2}gt^2$ — ②

From ① $t = \frac{x}{V\cos\theta}$

Substitute in ②

$y = V\times\frac{x}{V\cos\theta}\times\sin\theta - \frac{1}{2}g\left(\frac{x}{V\cos\theta}\right)^2$

$= x\tan\theta - \frac{1}{2}\frac{gx^2}{V^2\cos^2\theta}$

$= x\tan\theta - \frac{gx^2}{2V^2}\sec^2\theta$ ②

$= x\tan\theta - \frac{gx^2}{2V^2}(1+\tan^2\theta)$ — ③

(c) substitute $x = 240$, $y = 80$, $g = 10$ and $V = 60$ in ③

$80 = 240\tan\theta - \frac{10\times 240^2}{2\times 60^2}(1+\tan^2\theta)$

$80 = 240\tan\theta - 80(1+\tan^2\theta)$

$1 = 3\tan\theta - (1+\tan^2\theta)$

$3\tan\theta - (1+\tan^2\theta) = 1$

$\tan^2\theta - 3\tan\theta + 2 = 0$

$(\tan\theta-1)(\tan\theta-2) = 0$

$\tan\theta = 1$ or $\tan\theta = 2$ ③

Since $\beta < \alpha$

$\tan\beta = 1$ and $\tan\alpha = 2$

(ii) $x = Vt\cos\theta$

$240 = 60(t+T)\cos\alpha$

$240 = 60t\cos\beta$

$4 = (t+T)\cos\alpha$

$4 = t\cos\beta$

$\dfrac{4}{\cos\alpha} = t+T$

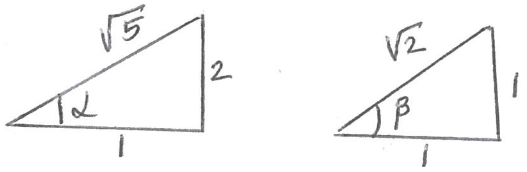
$\dfrac{4}{\cos\beta} = t$ (3)

$t+T = 4\sec\alpha$

$t = 4\sec\beta$

$T = 4\sec\alpha - 4\sec\beta$

$= 4(\sec\alpha - \sec\beta)$



$\cos\alpha = \dfrac{1}{\sqrt5}$ $\cos\beta = \dfrac{1}{\sqrt2}$

$\sec\alpha = \sqrt5$ $\sec\beta = \sqrt2$

$T = 4(\sqrt5 - \sqrt2)$ seconds

Question 14 (12 marks)

(a)(i) $y = x\cos^{-1}x - \sqrt{1-x^2}$

$\dfrac{dy}{dx} = x \times \dfrac{-1}{\sqrt{1-x^2}} + \cos^{-1}x - \dfrac{1}{2\sqrt{1-x^2}} \times -2x$ (2)

$= \dfrac{-x}{\sqrt{1-x^2}} + \cos^{-1}x + \dfrac{x}{\sqrt{1-x^2}} = \cos^{-1}x$

(ii) $\int_0^{\frac12} \cos^{-1}x\,dx = \left[x\cos^{-1}x - \sqrt{1-x^2}\right]_0^{\frac12}$

$= \left(\frac12\cos^{-1}\frac12 - \sqrt{1-\frac14}\right) - (0 - \sqrt1)$

$= \frac12 \times \frac{\pi}{3} - \sqrt{\frac34} + 1$ (2)

$= \frac{\pi}{6} - \frac{\sqrt3}{2} + 1$

(b) $P(x) = x^4 - 5x^3 - 9x^2 + 81x - 108$

$P'(x) = 4x^3 - 15x^2 - 18x + 81$

$P''(x) = 12x^2 - 30x - 18$

$P''(x) = 0$

$12x^2 - 30x - 18 = 0$

$2x^2 - 5x - 3 = 0$ (3)

$(x-3)(2x+1) = 0$

$x = 3$ or $x = -\frac12$

$P(3) = 81 - 135 - 81 + 243 - 108 = 0$

$x = 3$ is the triple root

Let the roots be $3, 3, 3$ and α

$\alpha + 9 = 5$; $\alpha = -4$

The roots are $3, 3, 3, -4$

(c) (i) $T = 25 + Ae^{-kt}$

LHS $= \frac{dT}{dt} = Ae^{-kt} \times -k$

$= -kAe^{-kt}$

RHS $= -k(T-25)$

$= -k \times Ae^{-kt}$

$= -kAe^{-kt}$ (2)

LHS = RHS

$\therefore T = 25 + Ae^{-kt}$ is a solution of $\frac{dT}{dt} = -k(T-25)$

(ii) when $t=10$, $T = 16°C$

$T(t) = 25 + Ae^{-kt}$

$T(0) = 10 = 25 + A$

$A = 10 - 25 = -15$

$16 = 25 - 15e^{-10k}$

$e^{-10k} = \frac{9}{15}$ (3)

when $t = 40$, $T = ?$

$T = 25 - 15e^{-40k}$

$= 25 - 15e^{4(-10k)}$

$= 25 - 15 \times \left(\frac{9}{15}\right)^4$

$= 23.056$

Question 15 (12 marks)

(a) $m(10m - 17) + 6 = 0$

$10m^2 - 17m + 6 = 0$

$10m^2 - 5b - 12b + 6 = 0$

$5b(2b-1) - 6(2b-1) = 0$ (3)

$(2b-1)(5b-6) = 0$

$b = \frac{1}{2}$ or $b = \frac{6}{5}$

(b) $\underset{\sim}{u} \cdot \underset{\sim}{w} = 2b + 2b = 4b$

$|\underset{\sim}{u}||\underset{\sim}{w}|\cos\theta = \sqrt{b^2+4} \cdot \sqrt{b^2+4} \cos 60$

$= (b^2 + 4) \times \frac{1}{2}$

$4b = (b^2+4) \times \frac{1}{2}$

$8b = b^2 + 4$

$b^2 - 8b + 4 = 0$

$b = \frac{8 \pm \sqrt{64 - 4 \times 1 \times 4}}{2}$

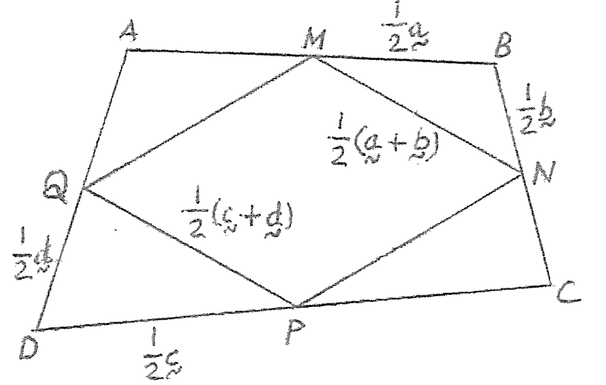
$= \frac{8 \pm \sqrt{48}}{2}$ (4)

$= \frac{8 \pm 4\sqrt{3}}{2}$

$= \frac{4(2 \pm \sqrt{3})}{2}$

$= 2(2 \pm \sqrt{3})$

(c) (i)



$a + b + c + d$
$= \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA}$
$= \overrightarrow{AC} + \overrightarrow{CD} + \overrightarrow{DA}$
$= \overrightarrow{AD} + \overrightarrow{DA} = 0$

(2)

(ii) $\overrightarrow{MN} = \overrightarrow{MB} + \overrightarrow{BN}$

$= \frac{1}{2}a + \frac{1}{2}b$

$= \frac{1}{2}(a + b)$

$\overrightarrow{PQ} = \overrightarrow{PD} + \overrightarrow{DQ}$

$= \frac{1}{2}c + \frac{1}{2}d$

$= \frac{1}{2}(c + d)$

(3)

But $a + b + c + d = 0$

$a + b = -(c + d)$

$\therefore MN = PQ$ and $MN \parallel PQ$

$\therefore$ MNPQ is a parallelogram.